INSTITUTE OF ACTUARIES OF INDIA

CM2 - Economic Modelling (Paper A)

November 2025 Examination

Indicative Solution

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

MCQ Solution 1 to 15 (2 marks each)

Solution 1 C) Stock prices fully reflect all available information, making it impossible to consistently achieve higher returns without taking on additional risk.

Solution 2 C) 2W(t) dW(t)+dt

Solution 3 D) Higher premium loading θ

Solution 4 C) Overestimation of the proportion reported

Solution 5 D) *The cumulative normal distribution of a negative d2 term from the Black-Scholes formula.*

Solution 6 A) *r_f is foreign interest rate; r_d is domestic interest rate*

Solution 7 A) Higher than the input volatility

Solution 8 B) EL $= PD \times LGD \times EAD = 0.02 \times 0.40 \times 1{,}000{,}000 = 8{,}000$

Solution 9 C) Diversifiable risk decreases, and total risk approaches non-diversifiable risk

Solution 10 D) $E[X_t]= \mu +(X_0- \mu)e^{-\theta t}$, $Var[X_t]= \sigma^2 /2\theta\ (1-e^{-2\theta t})$

Solution 11 C)

Explanation:
The two-state model assumes a borrower can default only once. Once default occurs, the process terminates.

Solution 12 B) $E[R_i] = R_f + \beta_i(E[R_m] - R_f)$

Solution 13 C) . $VaR_{10} = VaR_1 \times \sqrt{10}$

Solution 14 B) $E(R_p) = w_1 E(R_1) + w_2 E(R_2)$

Solution 15 C) $w_1^2\, \sigma_1^2 + w_2^2\, \sigma_2^2 + 2\, w_1\, w_2\, \rho\, \sigma_1\, \sigma_2$

Solution 16.

$X_1=3Z$,

$X_2=9Z$

- ⇨ $E[X_2|X_1=3]=9Z=3\cdot3=9$
- ⇨ So it's not a martingale because $E[X_2|F_1] \neq X_1$ **[3 marks]**

Solution 17.

Present value of dividend:

$D_{PV} = 2\cdot e^{-0.05.0.5} = 2\cdot e^{-0.025} \approx 2\cdot0.9753 = 1.9506$

Present value of strike:

$Ke^{-rT} = 105\cdot e^{-0.05} \approx 105\cdot0.9512 = 99.876$

Use put-call parity:

P=C + PV(K) + PV(D) − S

P = 9 + 99.876 + 1.9506 – 100 = 10.8266 **[3 marks]**

Solution 18.

U(x) = 10x - 0.5x^2

dU}/dx = 0=>10-x=0=>x=10

Check the second derivative:

$D^2U\}/dx^2=-1<0$, hence maximum **[3 marks]**

Solution 19

i) X = 72000/80000 – 1 = -10%

P(X<-10%) = P(Z<(-10% - 7%)/5.5%) = P(Z<-3.09) = 0.1% [2 marks]

ii) P(Z<(t-7%)/5.5%) = 0.005 \

(t-7%)/5.5%) = -2.5758

t = -7.1669%

80000 * (1-7.1669%) = ₹74266.48 [2 marks]

[4 Marks]

Solution 20.

The number of monthly claims N follows a binomial distribution: N ~ B(5000, 0.01), so

E(N) = np = 5000 x 0.01 = 50

Var(N) = npq = 5000 x 0.01 x 0.99 = 49.5

The individual claim amounts X are distributed uniformly between 0 and 10,000 so

E(X) = 0.5 x (0 + 10,000) = 5000

Var(X) = (1/12) x $(10{,}000 – 0)^2$ = 83,33,333.33

The monthly aggregate claim amount S then has

E(S) = E(N)E(X) = 50 x 5000= 2,50,000

Var(S) = E(N)Var(X) + Var(N)$E(X)^2$ = 1,65,41,66,666.67

SD(S) = 40,671.45 **[4 marks]**

Solution 21.

- Long-tailed business has claims that develop slowly over many years, with late reporting and payment lags since new developments comes into light as time passes.
- Reserving in these lines depends heavily on assumptions about future claim development which comes from development pattern. Development Patterns is Future Claim Expectations.
- Small changes in development patterns can have large financial impacts due to the long horizon and high uncertainty.
- The claims development pattern is derived from run-off triangles (e.g., using Chain-Ladder or Bornhuetter-Ferguson methods).
- Development pattern is proportion of claims expected to emerge in each future development year.

- But this implicitly assumes stability in claims and payment pattern over time — which may not hold in long-tailed lines.
- Development triangles for long-tailed lines often show that only a small percentage of claims are settled in early development years.
- In long-tailed business, later development years contribute most to reserves. This happens because of delayed claims emergence and Settlement.
- Claims may be reported years after the accident and Paid out over a long time horizon
- This means a significant portion of claims are still outstanding many years after the underwriting year.
- For example:
    - A short-tailed line like auto may have settled 90–95% of claims.
    - A long-tailed line may have settled 50–70%, meaning the rest (30–50%) is still in the tail.
- Even slight mis-specification in tail assumptions or extrapolation can lead to significant reserve inadequacy or over-reserving.
- Tail Has a High Weight in IBNR since a large portion of claims remains unsettled in later years, the reserve estimate depends heavily on accurate projection of the tail.
- Any error in tail estimation gets amplified, especially if claims are large or volatile.
- This is especially risky if:
    - There's limited historical data in the tail.
    - There have been structural changes (e.g., legal trends, inflation). **[4 marks]**

Solution 22.

Utility Function: U(x) = $\sqrt{x}$

Initial Wealth (a): ₹20,000

Random Loss (X): X ~ U(0, 20000)

PDF ($f_X(x)$): 1/ [20,000-0]=1/20000, 0<=x,=20,000

E[U(a-X)] = $\int_0^{20000} \frac{(20000-x)^{0.5}dx}{20000}$

=$[-2/(3*20000)*(20000-x)^{3/2}]_0^{20000}$

=2/3*20000^0.5=94.2809

The maximum premium P is found by solving the insurance equation:

E[U(a-X)] = U(a-P)

94.2809 = $(20000-P)^{0.5}$

=>8888.889=20000-P

=>P=11,111.11 **[5 marks]**

Solution 23.

i) Asset A Variance = $4^2 * npq$ = 23.04%%

Asset B Variance = $1.5^2 * \sigma^2$ = 36%% [2 marks]

ii) Asset A P(4X<3) = P(4X<3) = P(X≤0.75) = P(X=0)= 6C0 $0.4^0 * (1-0.4)^6$=0.046656

Asset B P(3Y<3) = P(Y<1) = P(Z<-1.1) = 0.13567 [4 marks]

[6 Marks]

Solution 24.

(20/160)*5% + (100/160)*10% + (40/160)*x = R_M

x – 3% = 2(R_M – 3%) => x = 2*R_M – 3%

=> 10*5% + 50*10% + 20*(2*R_M – 3%) = 80* R_M

=> R_M = (10*5% + 50*10% - 20*3%) / 40 = 12.25%

OR

(10% - 3%) / (R_M – 3%) = 1

=> R_M = 10%

Or full credit for any other valid approach.

For return on asset class C

x – 3% = 2*(12.25% - 3%)

=> x = 21.5%

OR

x – 3% = 2*(10% - 3%)

=> x = 17%

full credit for any other valid approach.

5% - 3% = β*(12.25% - 3%)

=> β = 0.216

OR

5% – 3% = β *(10% - 3%)

=> β = 0.286

Or full credit for any other approach **[6 marks]**

Solution 25.

Adjustment co-efficient R satisfies:

$\lambda ((E[e^{RX}]-1)=cR$

Claim size X ~ Exp (θ=1/2) since claim size X with mean $m_1 = 1/\theta=2 \Rightarrow \theta=1/2$

its MGF is

$E[e^{RX}]=\frac{1}{1-2R}$ for R<1/2

The security loading condition must be met: $c > \lambda m_1$ i.e 5>2*2 so this holds

Substitute the expressions into the fundamental equation:

$\lambda ((E[e^{RX}]-1)=cR$

$=> 2 * \left(\frac{1}{1-2R}\right) - 1$ =5R

$=>2 * \left(\frac{2R}{1-2R}\right)$=5R

$=>\left(\frac{4R}{1-2R}\right)$ =5R =>solve for R to get 5-10R=4, R=1/10=0.1

Apply Lundberg's inequality $\psi(u) \le e^{-Ru}$

$\psi(u) \le e^{-0.1*10} = e^{-1}=0.3679$ **[6 marks]**

Solution 26.

Let there be x shares and y units of cash. The value of hedging portfolio must be equal to the value of the call option at time t . This gives

120x+y= 5.50 (eq1)

Delta of hedging portfolio must be equal to delta of call option at time t. This gives

$x*\Delta_{share} + y*\Delta_{cash}=0.68$

$\Delta_{share}=1$, $\Delta_{cash}=0$, so x=0.68 , putting this value in eq 1

120*0.68+y=5.50 => y=-76.1

So hedging consists of buy 0.68 shares of value Rs 81.60 with borrowing cash of Rs 76.10

[6 marks]

Solution 27.

Use Put-Call Parity:

$C+Ke^{-rT}=P+S$

Plug in values to see if parity holds:

- C=12
- K=100
- $r=5\% \Rightarrow e^{-0.05.1}=e^{-0.05} \approx 0.9512$
- $Ke^{-rT}=100 \cdot 0.9512=95.12$
- P=10
- S=100

LHS (Call + PV of Strike):

$C+Ke^{-rT}$= 12 + 95.12 = 107.12

RHS (Put + Spot Price):

P+S=10+100=110

Hence LHS=107.12<RHS=110

So, the call is underpriced or the put is overpriced → Arbitrage opportunity exists.

Arbitrage Strategy (Buy low, sell high)

To exploit this:

1. Buy Call at ₹12

2. Sell Put at ₹10

3. Short the stock at ₹100

4. Invest PV(K) = ₹95.12 in risk-free bond

Initial cash flow (arbitrage profit)

- Receive:
  - ₹10 from selling put
  - ₹100 from short selling stock
- Pay:
  - ₹12 for buying call
  - ₹95.12 to buy zero-coupon bond

Net cash inflow = (10+100)−(12+95.12) = 110−107.12 = ₹2.88 **[6 marks]**

Solution 28.

$P(S_{0.5} > 110 \mid S_0 = 100)$

$= P(S_{0.5}/ S_0 > 11 /10)=P(\exp[(\mu-\sigma^2/2)/2+ \sigma B_{0.5}]>1.1)$

$=P((\mu-\sigma^2/2)/2+ \sigma B_{0.5}>\ln 1.1)$

Putting values of σ=10% and μ=20%

$P(1/2*(0.2-1/2*0.1^2)+0.1B_{0.5} > \text{Ln}1.1)= P(B_{0.5}>-0.022)$

Since $B_{0.5}$ is N(0, ½)

$= P(B_{0.5}>-0.022)= \Phi(0.031)=0.512= 51.2\%$ **[6 marks]**

Solution 29.

i) $E[1+i_t]= \exp(\mu+ \sigma^2/2)= 1.05$

$Var[1+i_t]= \exp(2\mu+ \sigma^2)*(\exp(\sigma^2)-1)=0.03^2=0.0009$

$E[1+i_t]^2= \exp(2\mu+ \sigma^2)$

$=>\exp(2\mu+ \sigma^2)*(\exp(\sigma^2)-1)=1.05^2*(\exp(\sigma^2)-1)= 0.0009$

$=> \sigma^2=\ln(0.0009/1.05^2+1)=0.00081599$

$=> \sigma=0.0285655$

$\exp(\mu+ 0.00081599/2)= 1.05$

$\mu=\ln(1.05)-(0.00081599)/2=0.0483822$ [4 marks]

ii) $P(0.01 < i_t < 0.03)$
$=P(1.01<1+ i_t< 1.03)$
$= P(\ln 1.01<\ln(1+ i_t)< \ln 1.03)$
$= P([\ln 1.01)- \mu]/ \sigma <[\ln(1+ i_t)- \mu]/ \sigma <[\ln 1.03- \mu]/ \sigma)$
Substituting the value of μ and σ from solution above
$= P(-1.3454 < Z < -0.6590) = 0.2549 - 0.0893 = 0.1656=16.56\%$ [4 marks]

[8 Marks]

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