INSTITUTE OF ACTUARIES OF INDIA

CM1 - Actuarial Mathematics for Modelling (Paper A)

November 2025 Examination

Indicative Solution

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

MCQ Question 1 to 15 (2 marks each)

Solution 1: Equivalent annual effective rate
Method: Quarterly rate = 0.078 / 4 = 0.0195.
$i_{eff} = (1 + 0.0195)^4 - 1$
$= (1.0195)^4 - 1$
$\approx 1.08031 - 1 = 0.08031$
= 8.03%.
Answer: Option D.

Solution 2: Relation between i and d
Method: By definition $d = i/(1+i)$. Rearranging gives $i = d/(1-d)$.
Answer: Option A.

Solution 3: Growing perpetuity-immediate
Method: PV = C1 / (i − g) = 1,000 / (0.05 − 0.02) = 33,333.33.
Answer: ₹33,000 (round)
Option B.

Solution 4: Project NPV
Method: $v=1/1.06$;
$a4 = (1 - v^4)/0.06$;
PV inflows = $15{,}000\times a4 + 10{,}000\times v^5$;
NPV = PV inflows − 50,000.
Computation: $v \approx 0.943396$, $v^4\approx0.792094$, $a4 \approx 3.46510$, PV inflows ≈ 59,449, NPV ≈ 9,449.
Answer: Option E

Solution 5: Quarterly loan repayment (6 years, annual i = 9%)
Method: $i_q=(1+0.09)^{(1/4)}-1$;
$n=24$;
$a24=(1-(1+i_q)^{-24})/i_q$;
R=60,000/a24.
Computation: $i_q\approx0.021778$, $a24\approx18.54$, $R\approx3{,}236$.
Answer: Option E.

Solution 6: IRR of project (−10,000; +4,000; +4,500; +5,500)
Method: Solve NPV=0 by trial and interpolation.
At 17% NPV≈+139; at 18% NPV≈−31.
Interpolate: IRR≈17.8%.
Answer: Option E.

Solution 7: Multiple decrement (constant forces $\mu^1=0.02$, $\mu^2=0.03$)
Method: $\mu=0.05$;
$q=1-e^{(-\mu)}$
$=1-e^{(-0.05)}=0.0488$.
Answer: Option A.

Solution 8: Probability at least one survives one year ($p_x=0.97$, $p_y=0.95$)
Calculation:
P(x survives) = 0.97,   P(y survives) = 0.95
Lives independent.
P(at least one survives) = 1 – P(both die)

= 1 – (0.03 × 0.05)
= 1 – 0.0015 = 0.9985
Final Answer: 0.9985 Option D

Solution 9: Decomposition of q_x=0.05 (60%/40%)
Method: q1=0.6×0.05=0.03;
q2=0.4×0.05=0.02.
Answer: (0.03, 0.02) Option A.

Solution 10: Prospective reserve at start of year 3 (P=₹952)
Method: V_2 = SA × v × q;
v=1/1.05=0.952381;
V_2=100,000×0.952381×0.01≈952.
Answer: Option B.

Solution 11: — Net single premium (5-year term, SA=100,000; q=0.01; i=6%)
NSP = SA * q * v * $(1-r^5)/(1-r)$, where v=1/(1+i), r=v(1−q)
v=1/1.06=0.9433962, r=0.93396, r^5≈0.71068
Factor=(1−0.7096425)/(1−0.9339623)=4.3819
NSP/SA = 0.009433 × 4.3819 ≈ 0.04133
NSP=100,000×0.04133 = ₹4,133
Answer: option A

Solution 12:
Non-unit reserves are to be held to zeroise all future negative cashflows except any occurring in the first policy year.

We are required to find: Non-unit reserve at the end of policy year 1.

Future (after year 1) negative cashflow = -₹200 (at end of year 2)

Let i = 5% = 0.05
Let V_1 = non-unit reserve at end of year 1

Reserve must accumulate with interest to meet ₹200 at end of year 2:

V_1 × (1 + i) = 200
V_1 = 200 / (1.05)
= 190.47619

Non-unit reserve required at the end of policy year 1 = ₹190
Answer: Option A

Solution 13:
Let v = 1/1.05 ≈ 0.952381.
Duration condition gives:
$(2Xv^2 + 6Yv^6) / (Xv^2 + Yv^6) = 4$
$Xv^2 = Yv^6$
$X/Y = v^4 = (1/1.05)^4$ ≈ 0.8227027 rounded to 0.823
Answer: Option C

Solution 14: Reserve recursion (V_2=400; P=1,000; SA=5,000; q=0.02; i=0.06)

$V_{(t+1)} = [(1 + i) \times (V_t + P) - q \times SA] / (1 - q)$
$V_3 = [(1.06) \times (400 + 1{,}000) - 0.02 \times 5{,}000] / (1 - 0.02)$
$V_3 = [1.06 \times 1{,}400 - 100] / 0.98$
$V_3 = [1{,}484 - 100] / 0.98$
$V_3 = 1{,}384 / 0.98$
$V_3 = 1{,}412.24$
Answer: Option E

Solution 15: Effect on reserve if valuation i < premium basis i
Method: Lower valuation → higher PV(liabilities) → higher reserve.
Answer: Option A.

Solution 16: Theory of Interest: Nominal vs Real Rates and Inflation,

i) Relationship between nominal(i), real(r), and inflation rates(j):
$1 + i = (1 + r)(1 + j)$
$\Rightarrow r = (1 + i)/(1 + j) - 1$ [1 mark]

ii) Real rates of interest:
Given nominal i = 6%.
For first 3 years, $j = 3\% \Rightarrow r_1 = 1.06/1.03 - 1 = 0.029126 \approx 2.9126\%$.
For next 2 years, $j = 4\% \Rightarrow r_2 = 1.06/1.04 - 1 = 0.019231 \approx 1.9231\%$. [1 mark]

iii) Real accumulated value after 5 years:
Real accumulation factor $= (1 + r_1)^3(1 + r_2)^2$
$= (1.06/1.03)^3(1.06/1.04)^2$
$= 1.13227$
Real accumulated value = 500,000 × 1.1322725 = ₹566,136.27. [1 mark]

iv) Comment:
- Real-return uncertainty: Variable inflation reduces certainty of real returns even if nominal yields are fixed;
- periods of higher inflation erode purchasing power and reduce real accumulation
- Duration / asset–liability matching: For long-term commitments, treasury should prefer instruments that protect real returns (inflation-linked bonds, TIPS/ILBs) or hedge inflation risk via derivatives or matching cash-flows.
- Policy implication: When inflation is expected to rise, shift towards inflation-indexed assets or shorten duration to reduce exposure; maintain contingency buffers and stress-test cashflows under alternate inflation scenarios.
- Practical note: For corporate treasury, set a real return target and choose nominal/inflation-linked mix to meet that target under plausible inflation paths.

[2 marks]

v) One-year forward rate for borrowing during year 3 (i.e., $f_{2,3}$)

Formula:
$(1 + s_3)^3 = (1 + s_2)^2 \times (1 + f_{2,3})$
$1 + f_{2,3} = (1 + s_3)^3 / (1 + s_2)^2$
$f_{2,3} = [(1 + s_3)^3 / (1 + s_2)^2] - 1$

Given:
$s_2 = 5.4\% = 0.054$
$s_3 = 6.1\% = 0.061$

Step-by-step:
$(1 + s_3)^3 = (1.061)^3 = 1.193053$
$(1 + s_2)^2 = (1.054)^2 = 1.110916$

$f_{2,3} = (1.193053 / 1.110916) - 1$
$= 1.07513978 - 1$
$= 0.07514 = 7.51\%$

Answer:
Forward rate for Year 3 = 7.51% p.a. (annual effective) [2 marks]

vi) Price of a 3-year 6% annual coupon bond (Face value ₹100)

Coupon = ₹6 per year, Redemption = ₹100 at end of Year 3

Price formula using spot rates:
$P = 6 / (1 + s_1) + 6 / (1 + s_2)^2 + 106 / (1 + s_3)^3$

Step-by-step:
1. $(1 + s_1) = 1.048 \rightarrow 6 / 1.048 = 5.7252$
2. $(1 + s_2)^2 = 1.110916 \rightarrow 6 / 1.110916 = 5.4017$
3. $(1 + s_3)^3 = 1.193053 \rightarrow 106 / 1.193053 = 88.7475$

$P = 5.7252 + 5.4017 + 88.7475 = 99.8744$

Answer:
Bond Price = ₹99.87 (slightly below par) [2 marks]

vii) The implied forward rate for Year 3 (7.51%) is higher than the current spot rates (4.8%, 5.4%, 6.1%), indicating an upward-sloping yield curve.

- Market expects higher short-term interest rates in future.
- EcoInvest should anticipate rising borrowing costs around Year 3.
- They may consider locking in longer-term funding or using hedging instruments to manage interest rate risk. [1 mark]

[10 Marks]

Solution 17 : Bond Pricing, Yield and Duration",

i) Price:
Equation of value:
$P = 8{,}000 \times a_{5|0.09} + 1{,}00{,}000 \times v^5$
where $v = 1 / (1 + i)$

$i = 0.09$; $v = 1 / 1.09 = 0.91743119$
$v^5 = (1.09)^\wedge(-5) = 0.6499311$
$a_5 = (1 - v^5) / i = (1 - 0.649931) / 0.09 = 3.889656$

Price
$P = 8,000 \times 3.889656 + 1,00,000 \times 0.649931$
$= 31,117.25 + 64,993.10$
$=$ ₹96,110.35 [4 marks]

ii)
Definitions:
$D = \Sigma[t \times CF_t \times v^t] / P$
$D_{mod} = D / (1 + i)$

Discount factors:
$v^1=0.917431$, $v^2=0.841682$, $v^3=0.772183$, $v^4=0.708429$, $v^5=0.649931$

Discounted cashflows:
t=1: $8,000 \times v=7,339.45$
t=2: $8,000 \times v^2=6,733.46$
t=3: $8,000 \times v^3=6,177.46$
t=4: $8,000 \times v^4=5,667.43$
t=5: $1,08,000 \times v^5=70,192.55$

Weighted ($t \times CF \times v^t$):
t=1: 7,339.45
t=2: 13,466.91
t=3: 18,532.39
t=4: 22,669.73
t=5: 3,50,962.74
Sum = 4,12,971.22

Macaulay Duration:
D = 4,12,971.22 / 96,110.35 = 4.2969 years

Modified Duration:
$D_{mod} = D / (1 + i) = 4.2969 / 1.09 = 3.942$ years [4 marks]

iii)
For $\Delta i = +0.005$ (0.5% rise):
$\Delta P/P \approx -D_{mod} \times \Delta i$
$= -3.942 \times 0.005 = -0.01971 = -1.971\%$

Estimated % price change = −1.97%
Approx. new price = $96,110 \times (1 - 0.01971) \approx$ ₹94,215 [2 marks]
[10 Marks]

Solution 18: Joint Life

i) $NSP = SA \times \Sigma_{k=1 \text{ to } 3} v^k [e^{(-\mu(k-1))} - e^{(-\mu k)}]$, where $\mu = \mu_m + \mu_f$

$\mu = 0.030 + 0.025 = 0.055$
$i = 0.075 \rightarrow v = 1/(1+0.075) = 0.93023$

Thus,
$NSP = SA \times [v(e^0 - e^{(-\mu)}) + v^2(e^{(-\mu)} - e^{(-2\mu)}) + v^3(e^{(-2\mu)} - e^{(-3\mu)})]$ [3 marks]

ii)
Compute e^(−μt):
e^(−μ) = e^(−0.055) = 0.946485
e^(−2μ) = e^(−0.11) = 0.895834
e^(−3μ) = e^(−0.165) = 0.84789

First-death probabilities by year:
Year 1: 1 − e^(−μ) = 0.053515
Year 2: e^(−μ) − e^(−2μ) = 0.050651
Year 3: e^(−2μ) − e^(−3μ) = 0.047940

Discount factors:
$v^1 = 0.93023$, $v^2 = 0.86533$, $v^3 = 0.80496$

Discounted probabilities and contributions:
Year 1: 0.93023 × 0.053515 = 0.04978 → 250,000 × 0.04978 = ₹12,445.31
Year 2: 0.86533 × 0.050651 = 0.04383 → 250,000 × 0.04382 = ₹10,957.46
Year 3: 0.80496 × 0.047940 = 0.03859 → 250,000 × 0.03859 = ₹9,647.446

Total discounted probability = 0.132201

NSP = 250,000 × 0.132201 = ₹33,050.22

NSP ≈ ₹33,050 Rounded to nearest rupee [5 marks]

iii)
Under positive dependence (e.g. common lifestyle or shared environmental risk), the probability that both survive to a given time increases relative to independence, reducing the likelihood of a *single* early death.

Hence, the expected first-death probability decreases, and the NSP would be lower than under independence.

If, however, the dependence arises from a strong common-shock model (e.g. simultaneous deaths from accidents), the timing of payments could shift earlier, potentially increasing the NSP.

The overall effect depends on the precise dependence structure, and assumptions used.

[2 marks]
[10 Marks]

Solution 19: 10-year Endowment (Net Annual Premium),

i)
EPV(Premiums) = EPV(Benefits) + EPV(Expenses)
Or
The equivalence principle states that the expected present value of all future premiums must equal the expected present value of all future benefits and expenses, ensuring that the contract is priced fairly at inception. [1 mark]

ii) Derive and compute the annual premium

Let:

Sum Assured S=200,000
Term n=10 years
Interest i=0.06; v=1/(1.06)=0.943396
Mortality basis: AM92 Select (entry age 30)

EPV(Benefits)$=S\times\sum_{\{t=1to10\}}v^t({}_{t-1}p_{[30]})q_{([30]+t-1)}+S\times v^{10}({}_{10}p_{[30]})$

EPV(P) $=P\times\sum_{\{t=0to9\}}v^t{}_tp_{[30]}$

EPV(Exp)=0

$P\ddot{a}_{30:10}=S*A_{30:10}$

$A_{[30]:10} = A_{[30]} - (v^{10} \times {}_{10}p_{[30]} \times A_{40}) + v^{10} \times {}_{10}p_{[30]}$

where

$A_{[30]} = 0.07316$ (whole life assurance, AM92 Select, 6%)

$A_{40} = 0.12313$ (whole life assurance at age 40)

$v^{10} = (1.06)^{\wedge}(-10) = 0.558395$

${}_{10}p_{[30]} = 0.9932$

$A_{[30]:10} = 0.55947$

$\ddot{a}_{30:10} = \ddot{a}_{30} - v^{10}{}_{10}p_{[30]}\,\ddot{a}_{40} = 7.7827$

P=14,377 [7 marks]

iii) Practical pricing adjustments

Adjustment 1: Expense loading:In practice, a loading is added to recover both initial acquisition expenses (medical, underwriting, commission) and renewal expenses (administration, collection). These can be modeled as additional EPV(Expenses) in the equivalence equation or expressed as per-premium additions.

Adjustment 2: Profit and contingency margin: A profit margin or contingency reserve is often added by increasing the calculated net premium by a factor 1+λ, where λ\lambda reflects the insurer's target return or risk allowance.

This ensures solvency and cushions against adverse mortality or expense deviations.

[2 marks]
[10 Marks]

Solution 20: Reserving (Prospective & Retrospective),

i) Prospective reserve at time t = EPV at time t of future benefits and expenses less EPV at time t of future premiums:

$V_t^{pros}=EPV_t(\text{future benefits + Expenses})-EPV_t(\text{future premiums})$

Retrospective reserve at time t = accumulated value at time t of past premiums less accumulated value at time t of past benefits and past expenses:

V_t^{retro}=accumulated (past premiums)−accumulated (past benefits + expenses). ½ marks

They are equal (for all t) when the same mortality and interest assumptions are used and no arbitrage / no mismatch in conventions -

i.e. under the equivalence principle (consistent basis) the prospective and retrospective reserves coincide.

Alternative:

Definition of Prospective Reserve as per the core reading is:
The expected present value of the future outgo
Less
the expected present value of the future income.

Definition of Retrospective Reserve as per the Core reading is as follows:
The accumulated value allowing for interest and survivorship of the premiums received to date
Less
the accumulated value allowing for interest and survivorship of the benefits and expenses paid to date.

Equality of Prospective and Retrospective Reserves (as per Core reading):
If

1. the retrospective and prospective reserves are calculated on the same basis
2. this basis is the same as the basis used to calculate the premiums used in the reserve calculation, then the retrospective reserve will be equal to the prospective reserve.

[2 marks]

ii)
Equation of value:
$V_2 = 1{,}00{,}000 \times A_3|_{0.05} - 580 \times (v\,p + v^2\,p^2)$
where $A_3 = q\,v + q\,p\,v^2 + q\,p^2\,v^3$, $v = 1 / (1 + i)$, $p = 1 - q$

$i = 0.05 \quad v = 1 / 1.05 = 0.95238$

$q = 0.004$, $p = 0.996$
$pv = 0.996 \times 0.95238 = 0.94857$

$(pv)^2 = 0.94857^2 = 0.89979$
$A_3|_{0.05} = 0.004 \times 0.95238 \times (1 + 0.94857 + 0.89979)$
$= 0.0038 \times 2.84836$
≈ 0.01085

EPV of benefits = 1,00,000 × 0.01085089 = ₹1,085.09

PV of remaining premiums = 580 × (0.94857 + 0.89976)

= 580 × 1.84836
= ₹1,072.05 (rounded)

Reserve
$V_2 = 1{,}085.09 - 1{,}072.05 = +₹13.04$ [6 marks]

iii)
When the assumed valuation interest rate decreases, future policy benefits are discounted at a lower rate, leading to higher present values and therefore higher mathematical reserves.

Since the insurer's asset value usually remains broadly the same, this increase in reserves reduces the excess of assets over liabilities.

Thus, solvency margins decline when interest rates fall. [2 marks]
[10 Marks]

Solution 21:
i)
a)
Equation of value:
$NPV = -10 + (1.5 / 1.08^1) + (2.0 / 1.08^2) + (2.5 / 1.08^3) + (3.0 / 1.08^4) + (4.5 / 1.08^5)$
$=-10+1.3889+1.71468+1.98458+2.20509+3.06262$
= ₹0.3558 crore [2 marks]

b)
Cumulative inflows (₹ crore):
Year 1 = 1.5
Year 2 = 3.5
Year 3 = 6.0
Year 4 = 9.0
Year 5 = 12.5

Payback occurs between Year 4 and Year 5
Payback = 4 + (10 – 9) / 4.5 = 4.22 years [1 mark]

c)
Cumulative discounted cashflows (at 8%):
Year 1 = 1.39
Year 2 = 3.10
Year 3 = 5.08
Year 4 = 7.29
Year 5 = 10.356

Crosses 10 crore between Year 4 and 5
Discounted Payback = 4 + (10 – 7.29) / 3.066 = 4.88 years [2 marks]

ii)
Trial 1: At 8% NPV = +₹0.3559 crore
Trial 2: At 10% NPV = -₹0.262 crore

$IRR \approx r_1 + [NPV_1 \times (r_2 - r_1)] / (NPV_1 - NPV_2)$

$IRR = 0.08 + [0.3559 \times 0.02] / (0.3559 - (-0.2620))$

IRR = 0.08 + 0.00712 / 0.6179
IRR = 0.0913 [2 marks]

iii)
Step 1: Compute basic values
$v = 1 / (1 + i) = 1 / 1.0175 = 0.9828$
$d = i / (1 + i) = 0.017199$

Step 2: Compute v^6, v^{22}, and annuity-due factor
$v^6 = 0.901143$
$v^{22} = 0.682720$
$\ddot{a}_{22} = (1 - v^{22}) / d = (1 - 0.6827202841) / 0.0171990172 = 18.448$

Step 3: Present value equation
$L = P \times v^6 \times \ddot{a}_{22}$
$\Rightarrow P = L / (v^6 \times \ddot{a}_{22})$

Step 4: Substitute
$P = 50{,}000{,}000 / (0.9011425417 \times 18.44754919)$
P = ₹3,007,722.98

First instalment (quarterly in advance)
P = ₹3,007,722.98 [3 marks]

iv)
Year 3 corresponds to quarters 9–12 (t = 8,9,10,11).
At t = 8, opening balance = P*× ä20 = 51269816.61

Quarter	Opening Balance (₹)	Payment (₹)	after payment	interest	principal	Closing Balance (₹)
8	51269816.61	3007722.98	48262093.6	844586.64	2163136	49106680.26
9	49106680.26	3007722.98	46098957.3	806731.75	2200991	46905689.03
10	46905689.03	3007722.98	43897966	768214.41	2239509	44666180.45
11	44666180.45	3007722.98	41658457.5	729023.01	2278700	42387480.47

[5 marks]

v)
"End of 5th year" = time t = 20 (i.e. 5 years × 4 quarters)

Balances from amortisation:
Opening balance at t = 20
= P × ä8
= ₹22,662,114.09 [1 mark]

vi) How Profit Testing Supports Pricing & Capital Allocation
- Quantifies expected profit under realistic assumptions (mortality, expenses, investment returns).
- Tests sensitivity to key assumptions (interest, lapses, expenses) to identify model risks.
- Provides scenario and stress testing (best/worst/base) to assess capital adequacy under different outcomes.
- Informs product-level capital charges and economic capital allocation (risk-adjusted returns).

- Guides setting of expense and risk loadings to achieve target ROE and solvency buffers.
- Helps determine reinsurance needs and pricing to reduce capital strain or tail risk.
- Supports product design choices (term, guarantees, surrender terms) by showing profit drivers.
- Enables monitoring of in-force profitability over time and triggers for remedial action (price changes, expense control).
- Supplies management and ALM teams with cashflow projections to align asset strategy with liabilities.
- Provides transparent metrics (NPV, IRR, PI) for comparison across product lines and investment opportunities.

[2 marks]

vii)

- Market demand: Growing investor interest in ESG products can increase sales and customer acquisition.
- Brand differentiation: ESG link enhances brand positioning and helps attract younger demographics.
- Regulatory & compliance fit: Product aligns with regulatory emphasis on sustainable finance (reduces regulatory friction).
- Distribution readiness: Existing distribution channels and advisors trained on ESG can accelerate adoption.
- Operational capability: Back-office systems, reporting, and ESG fund managers are available to implement the product.
- Reputational upside: Demonstrable ESG commitments can strengthen stakeholder trust and long-term retention.
- Competitive landscape: Early mover advantage in ESG-linked life products may capture market share.
- Risk mitigation: Ability to hedge or reinsure specific product risks lowers capital requirements and downside exposure.

[2 marks]

[20 Marks]

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