INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

November 2025

CM2 - Economic Modelling (Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Q.1) Which of the following best describes the Efficient Market Hypothesis (EMH)?

A) Investors can consistently outperform the market by analysing past stock prices and trends.
B) All investors have access to inside information, which makes markets efficient.
C) Stock prices fully reflect all available information, making it impossible to consistently achieve higher returns without taking on additional risk.
D) Markets are inefficient and stock prices are always mispriced, allowing skilled investors to always beat the market.
E) All of the above

[2]

Q.2) Let $X(t)=W(t)^2$, where W(t) is a standard Brownian motion. According to Ito's Lemma, what is the differential dX(t)?

A) 2W(t)dt
B) 2W(t)dW(t)
C) 2W(t) dW(t) + dt
D) $dW(t)^2=0$, so dX(t)=2W(t) dW(t)
E) All of the above

[2]

Q.3) Which of the following factors will decrease the ultimate ruin probability?

A) Lower initial surplus u
B) Higher claim frequency λ
C) Larger average claim size $E[X]$
D) Higher premium loading θ
E) Larger claim size variance

[2]

Q.4) Which of the following would most likely cause the Bornhuetter–Ferguson reserve to be *understated*?

A) Overestimation of the expected loss ratio
B) Underestimation of the expected loss ratio
C) Overestimation of the proportion reported
D) Decrease in the premium volume
E) Overestimation of claim frequency

[2]

Q.5) In the Merton model, the risk-neutral probability of default over a horizon T is equivalent to:

A) The probability that the firm's equity becomes negative before maturity.
B) The probability that the firm's asset value at maturity falls below the risk-free bond value, computed under the real-world measure.
C) The value of a European put option on the firm's assets with strike equal to the face value of debt.
D) The cumulative normal distribution of a negative d_2 term from the Black-Scholes formula.
E) None of the above

[2]

Q.6) In the Garman-Kohlhagen model, the formula for the call option price is:

$$C = S_0 e^{-r_f T} N(d_1) - K e^{-r_d T} N(d_2)$$

What do the terms r_f and r_d represent?

A) r_f is foreign interest rate; r_d is domestic interest rate
B) r_f is domestic interest rate; r_d is inflation rate
C) r_f and r_d are both foreign interest rates
D) r_f is forward premium; r_d is spot premium
E) None of the above

[2]

Q.7) If the market price of an option is higher than the Black–Scholes theoretical price (using a given volatility), then the implied volatility is:

A) Higher than the input volatility
B) Lower than the input volatility
C) Equal to the input volatility
D) Always 0
E) Independent of volatility

[2]

Q.8) If the probability of default (PD) = 2%, loss given default (LGD) = 40%, and exposure at default (EAD) = ₹1,000,000, then expected loss (EL) = ?

A. ₹4,000
B. ₹8,000
C. ₹20,000
D. ₹40,000
E. ₹80,000

[2]

Q.9) What happens to total portfolio risk as the number of assets in a portfolio increases, assuming assets are not perfectly correlated?

A) Total risk remains constant regardless of the number of assets
B) Non-diversifiable risk increases while diversifiable risk decreases
C) Diversifiable risk decreases, and total risk approaches non-diversifiable risk
D) Total risk increases because more assets mean more exposure
E) Diversifiable risk increases, and total risk is diversifiable risk only

[2]

Q.10) For the Ornstein-Uhlenbeck process, $dXt = \theta(\mu - X_t)dt + \sigma\, dW_t$, the expected value and variance of X_t, assuming X_0 at t=0, are:

A) $E[X_t]= X_0$, $Var[X_t]= \sigma^2 t$
B) $E[X_t]= \mu$, $Var[X_t]= 0$
C) $E[X_t]= X_0 e^{-\theta t}$, $Var[X_t]= \sigma^2 e^{-2\theta t}$
D) $E[X_t]= \mu + (X_0 - \mu) e^{-\theta t}$, $Var[X_t]= \sigma^2 /2\, \theta\, (1-e^{-2\theta t})$
E) None of the above

[2]

Q.11) Which of the following is *not* an assumption of the standard two-state model?

A) Only two possible states (default or survival)
B) Constant default intensity

C) Default can occur multiple times
D) Independent defaults (in basic form)
E) Exponential default time distribution

[2]

Q.12) Which of the following correctly states the Sharpe-Linter CAPM equation for expected return?

A) $E[R_i] = R_f + \sigma_i(E[R_m] - R_f)$
B) $E[R_i] = R_f + \beta_i(E[R_m] - R_f)$
C) $E[R_i] = R_f + \rho_{im} * \sigma_i * \sigma_m$
D) $E[R_i] = \mu + \theta\beta_i$
E) None of the above

[2]

Q.13) The 10-day 99% VaR (assuming iid normal daily returns) is scaled from 1-day VaR using:

A) $VaR_{10} = \frac{VaR_1}{\sqrt{10}}$
B) $VaR_{10} = VaR_1 \times 10$
C) $VaR_{10} = VaR_1 \times \sqrt{10}$
D) $VaR_{10} = VaR_1 + 10$
E) $VaR_{10} = \frac{VaR_1}{10}$

[2]

Q.14) The expected return of a portfolio is calculated as:

A) $E(R_p) = \sqrt{w_1 E(R_1)^2 + w_2 E(R_2)^2}$
B) $E(R_p) = w_1 E(R_1) + w_2 E(R_2)$
C) $E(R_p) = E(R_1) \times E(R_2)$
D) $E(R_p) = w_1 + w_2 + E(R_1 + R_2)$
E) $E(R_p) = E(R_1 - R_2)$

[2]

Q.15) For a two-asset portfolio with weights w_1 and w_2, variances σ_1^2, σ_2^2, and correlation ρ, the portfolio variance is:

A) $w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2$
B) $\sigma_1^2 + \sigma_2^2 + 2 \rho \sigma_1 \sigma_2$
C) $w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho \sigma_1 \sigma_2$
D) $(w_1 \sigma_1 + w_2 \sigma_2)/2$
E) None of the above

[2]

Q.16) Let $X_n=3^n \cdot Z$, where $Z \sim N(0,1)$ is independent of n. Is $\{X_n\}$ a martingale? If not, what is $E[X_2|F_1]$ given $X_1=3$? **[3]**

Q.17) A European call and put option have the following parameters:

- Current stock price S=100
- Call price C=9
- Strike price K=105

- Time to maturity T=1 year
- Risk-free rate r=5% (compounded continuously)
- The stock will pay a ₹2 dividend in 6 months

What is the price of the European put option (P)? **[3]**

Q.18) An individual has a quadratic utility function given by:

$U(x)=ax-bx^2$

where a=10, b=0.5, and x is the quantity of a good consumed.

What is the optimal consumption level x^* that maximizes utility? **[3]**

Q.19) An investor has ₹80,000 to invest, for a period of 1 year, and has identified two investment opportunities in which to invest. The first is a direct investment in a stock index for a period of 1 year. The annual return, X, on the index follows a Normal distribution with mean μ = 7% p.a. and standard deviation σ = 5.5% p.a.

Calculate the following in respect of the investment at the end of 1 year:

i) The shortfall probability below a value of ₹72,000 (2)

ii) The 99.5% value at risk. (2)

[4]

Q.20) An insurance company has written 5000 1-year car insurance policies. Each policy charges a monthly premium of ₹100 over the life of the policy. You can assume for your calculations that the premium is paid continuously. The probability of a claim on each policy in any given month is 1%, and claims are accounted for on the last day of each month. Claim amounts follow a uniform distribution between ₹0 and ₹10,000. Claims are independent and each policy can have at most one claim per month.

Calculate the mean and standard deviation of the monthly aggregate claims. **[4]**

Q.21) Mr. XYZ, an actuary, is looking into the long-tailed lines of business and worried on the choice of development pattern.

To what extent does the choice of development pattern impact reserve adequacy for long-tailed lines of business? **[4]**

Q.22) The maximum premium, P, which an individual will be prepared to pay in order to insure themselves against a random loss X is given by the solution of the equation:

E[U(a-X)] = U(a-P)

where a is the initial level of wealth.

Consider an individual with a utility function of $U(x) = \sqrt{x}$ and current wealth of ₹20,000. Assume that this individual is at risk of suffering damages that are uniformly distributed up to 20,000.

Calculate the individual's expected utility and the maximum premium P. **[5]**

Q.23) Two assets are available for investment. Asset A returns 4X%, where X is a Binomial random variable with parameters n = 6 and p = 0.4. Asset B returns 3Y%, where Y is a Normal random variable with parameters μ = 3.2 and σ = 2. Calculate the following separately for each of asset's A and B:

i) The variance of the return (2)

ii) The shortfall probability below a return of 3%. (4)

[6]

Q.24) Consider the following assets in a world where the Capital Asset Pricing Model (CAPM) holds. There are three risky assets and one risk-free asset. No other assets exist in the market.

Asset	Expected Return (% pa)	Total value of assets in market (₹m)	Beta
Risky asset A	5	20	β
Risky asset B	10	100	1
Risky asset C	x	40	2
Risk-free asset	3	80	n/a

Calculate the expected return on the market portfolio and Expected Return on Asset class C and β . **[6]**

Q.25) An insurance company's surplus process follows the classical Cramér–Lundberg model, where:

- Initial surplus u=10
- Premium rate per unit time c=5
- Claims follow a Poisson process with intensity λ=2 claims per unit time
- Claim sizes are exponentially distributed with mean 2

Assuming the adjustment coefficient R exists, compute the approximate probability of ruin ψ(u) using the Lundberg inequality:

$$\psi(u) \leq e^{-Ru}$$

What is the numerical upper bound for the probability of ruin? **[6]**

Q.26) A call option has a price of ₹5.50 and a delta of 0.68 at time t. Determine the hedging portfolio of shares and cash for this option at time t, given that the price of underlying share S_t = ₹120. **[6]**

Q.27) A European call and put option have the following details:

- Current stock price (S) = ₹100
- Strike price (K) = ₹100
- Time to maturity (T) = 1 year
- Risk-free interest rate (r) = 5% per annum (continuously compounded)
- Call option price (C) = ₹12
- Put option price (P) = ₹10

Is there an arbitrage opportunity? If yes, what is the arbitrage profit per share? **[6]**

Q.28) S_t , the price of a share at time t, is modelled as geometric Brownian motion. If μ = 20% pa and σ = 10% pa, calculate the probability that the share price will exceed 110 in six months' time given that its current price is 100.

$$S_t = S_0\, e^{(\mu - ½\sigma^2)t + \sigma B_t}$$

One of the properties of the geometric Brownian motion is:

The distribution of the increments $B_t - B_s$ $(0 \leq s < t)$ is given by:

$B_t - B_s \sim N(0, t-s)$

[6]

Q.29) A fund earns an annual rate of return" i_t", with the rate of return in any year being independent of the rate in any other year. The distribution of $\log(1 + i_t)$ is normal with parameters μ and σ^2. The mean of it is 5% and the standard deviation is 3%.

i) Calculate μ and σ^2. (4)

ii) Using information above, calculate the probability that the fund return for any year is between 1% and 3%. (4)

[8]

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