13th TechTalk on Employee Benefits

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Performance Matters: Valuing ESOPs with market-linked vesting conditions

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Today's agenda

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- Vanilla ESOPs
- Types of Vesting Conditions
- Accounting Standard Requirements
- Methods to Value ESOPs
- Pitfalls of Simulation techniques
- Example of a Market-Linked Vesting Condition
- Applying the First Principles Approach

Notations used in this presentation

t	Time
S_t	Share price at time t
K	Strike price/ Exercise price
T	Vesting date/ Exercise date/ Term of option
r	Continuously compounded risk-free rate (% p.a.)
q	Dividend yield (% p.a.)
v	Annualised standard deviation of daily returns (% p.a.)
ln	The natural logarithm function i.e., log with the base e (Euler's number)
Φ	The CDF of the standard Normal distribution i.e., CDF of N (0, 1)

Poll question

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Often ESOPs vest only after a few years of service is rendered by the employee. However, do you think ESOPs can be offered such that they only vest if the share price grows by, say, 25% p.a. over the vesting period?

A: Yes

B: No

Vanilla ESOPs

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Description	Aims	Features
European call option Gives the holder the right (but not the obligation) to purchase the underlying share at the specified price on the specified date	Retain employees Solve the ‘Principal-Agent problem’	Issued for a medium term (3 – 5 years) Vesting based on service Equity-settled “Pays” MAX $(S_T - K, 0)$

Types of Vesting Conditions

Vesting conditions

- Service conditions
  - Graded vesting
  - Bullet vesting
- Performance based conditions
  - Non-market linked e.g., be rated ‘Exceeds Expectations’
  - **Market-linked**

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Accounting Standard Requirements

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For non-market linked vesting conditions	For market linked vesting conditions
Not allowed for in the fair value of option granted	Allowed for in the fair value of option granted
Allowed for by adjusting number of options expected to vest	No 'true up' needed
Requires 'true up'	*Refer: Ind AS 102 ¶ 21*
Refer: Ind AS 102 ¶ 19 & ¶ 20	

Methods to value ESOPs

- Black-Scholes Option Pricing Model
- Binomial Trees
- Simulation

Pitfalls of Simulation Techniques

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Requires significant computational resource ($\geq$ 50,000 runs)

Validation is difficult

Can be a 'black box' model

Possible inconsistency of output with other techniques

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Example of a Market-Linked Vesting Condition

Co. X has ambitious growth plans and hence granted ESOPs to its employees where the proportion of ESOPs that finally vests depend on the growth in share price (i_1, i_2 and i_3) over the option term as described below

Scenario	Proportion of vesting	Payoff in scenario
If $S_T \geq X$	A%	A% × MAX (S_T – K, 0)
If $X > S_T \geq Y$	B%	B% × MAX (S_T – K, 0)
If $Y > S_T \geq Z$	C%	C% × MAX (S_T – K, 0)
IF $S_T < Z$	D%	D% × MAX (S_T – K, 0)

Where:

$X = S_0 \times (1+i_1)^T$

$Y = S_0 \times (1+i_2)^T$

$Z = S_0 \times (1+i_3)^T$

$X > Y > Z > K$

$A > B > C > D > 0$

Poll question

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How would you value this ESOP?

A	Use simulation to predict the final share price and determine the number of ESOPs that vest and multiply that with the payoff in that run
B	Use the Black-Scholes formula to value the payoff at expiry and calculate the probability of each scenario occurring and multiply the two to arrive at the fair value of the option
C	Use the Black-Scholes formula to value the payoff at expiry and ignore the vesting criterion

Example of a Market-Linked Vesting Condition

Determine the fair value of ESOPs as per the Black-Scholes formula (A)

Determine the probability of each scenario occurring (B)

Fair value of ESOP will be A × B

THIS DOUBLE COUNTS THE PROBABILITIES UNDERSTATE COSTS

Example of a Market-Linked Vesting Condition

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Black-Scholes formula already allows for probability of share price taking a particular value

Black-Scholes Formula = $\int_0^{\infty} e^{-rT} \max(S_T - K, 0).\, f(S_T).\, dS_T$

Multiplying this output with probability of scenario occurring double counts the probability!

Applying the First Principles Approach

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Fair value of ESOP = EPV of Payoff at Expiry

Key assumptions

- No arbitrage
- Share prices follow a Geometric Brownian Motion
- Options are exercised only at expiry
- The risk-free rate and volatility of share price are known constants

Applying the First Principles Approach

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$$S_T = S \mid S_0 \sim \text{Log N}\,[\mu = \ln S_0 + (r - q - ½v^2)\,T,\ \sigma^2 = v^2 T]$$

$$\text{PDF } f(s) = \frac{1}{\sigma\sqrt{2\pi}}\frac{1}{s}\,e^{-\frac{1}{2}\left(\frac{\ln s-\mu}{\sigma}\right)^2}$$

Applying the First Principles Approach

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Fair Value = EPV of Payoff at expiry

$$= e^{-rT} E[\text{ Payoff based on scenario } | S_0]$$

Scenario		Proportion of vesting	Payoff in scenario
If $S_T \geq X$	$[X = S_0 \times (1+i_1)^T]$	A%	A% × MAX (S_T – K, 0)
If $X > S_T \geq Y$	$[Y = S_0 \times (1+i_2)^T]$	B%	B% × MAX (S_T – K, 0)
If $Y > S_T \geq Z$	$[Z = S_0 \times (1+i_3)^T]$	C%	C% × MAX (S_T – K, 0)
IF $S_T < Z$	$[Z = S_0 \times (1+i_3)^T]$	D%	D% × MAX (S_T – K, 0)

$$= e^{-rT}\Big[A\% \int_X^{\infty} \max(S-K,0).f(s).ds + B\% \int_Y^{X} \max(S-K,0).f(s).ds +$$

$$C\% \int_Z^{Y} \max(S-K,0).f(s).ds + D\% \int_0^{Z} \max(S-K,0).f(s).ds\Big]$$

Applying the First Principles Approach

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Since X >Y > Z > K, we re-write the equation as:

$$= e^{-rT}\left[A\% \int_X^{\infty}(S-K).f(s).ds + B\% \int_Y^{X}(S-K).f(s).ds + V\% \int_Z^{Y}(S-K).f(s).ds + D\% \int_K^{Z}(S-K).f(s).ds\right]$$

Expanding the brackets & re-arranging the terms to group similar integrals together:

$$= e^{-rT}\left[A\% \int_X^{\infty} S.f(s)ds + B\% \int_Y^{X} S.f(s)ds + C\% \int_Z^{Y} S.f(s)ds + D\% \int_K^{Z} S.f(s)ds - K\left(A\% \int_X^{\infty} f(s)ds + B\% \int_Y^{X} f(s)ds + C\% \int_Z^{Y} f(s)ds + D\% \int_K^{Z} f(s)ds\right)\right]$$

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$$=e^{-rT}\left[A\%\int_X^{\infty} S.f(s)ds + B\%\int_Y^{X} S.f(s)ds + C\%\int_Z^{Y} S.f(s)ds + D\%\int_K^{Z} S.f(s)ds - K\left(A\%\int_X^{\infty} f(s)ds + B\%\int_Y^{X} f(s)ds + C\%\int_Z^{Y} f(s)ds + D\%\int_K^{Z} f(s)ds\right)\right]$$

Using the truncated moments of Lognormal distribution for the purple integrals

$$\int_L^U S.f(s)ds = e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(U)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(L)-\mu}{\sigma}-\sigma\right)\right]$$

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$$=e^{-rT}\left[A\%\int_X^\infty S.f(s)ds + B\%\int_Y^X S.f(s)ds + C\%\int_Z^Y S.f(s)ds + D\%\int_K^Z S.f(s)ds - K\left(A\%\int_X^\infty f(s)ds + B\%\int_Y^X f(s)ds + C\%\int_Z^Y f(s)ds + D\%\int_K^Z f(s)ds\right)\right]$$

The maroon integrals are

$$\int_L^U f(s)ds = \Phi\left(\frac{\ln U - \mu}{\sigma}\right) - \Phi\left(\frac{\ln L - \mu}{\sigma}\right)$$

Applying the First Principles Approach

$$\text{Fair value} = e^{-rT}\Bigg[A\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(\infty)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(X)-\mu}{\sigma}-\sigma\right)\right]+$$

$$B\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(X)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(Y)-\mu}{\sigma}-\sigma\right)\right]+$$

$$C\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(Y)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(Z)-\mu}{\sigma}-\sigma\right)\right]+$$

$$D\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(Z)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(K)-\mu}{\sigma}-\sigma\right)\right]-$$

$$K\Bigg(A\%\left\{\Phi\left(\frac{\ln\infty-\mu}{\sigma}\right)-\Phi\left(\frac{\ln X-\mu}{\sigma}\right)\right\}+B\%\left\{\Phi\left(\frac{\ln X-\mu}{\sigma}\right)-\right.$$

$$\left.\Phi\left(\frac{\ln Y-\mu}{\sigma}\right)\right\}+C\%\left\{\Phi\left(\frac{\ln Y-\mu}{\sigma}\right)-\Phi\left(\frac{\ln Z-\mu}{\sigma}\right)\right\}+D\%\left\{\Phi\left(\frac{\ln Z-\mu}{\sigma}\right)-\right.$$

$$\left.\Phi\left(\frac{\ln K-\mu}{\sigma}\right)\right\}\Bigg)\Bigg]$$

Applying the First Principles Approach

$$\text{Fair value} = e^{-rT}\Bigg[A\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(\infty)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(X)-\mu}{\sigma}-\sigma\right)\right]+$$

$$B\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(X)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(Y)-\mu}{\sigma}-\sigma\right)\right]+$$

$$C\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(Y)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(Z)-\mu}{\sigma}-\sigma\right)\right]+$$

$$D\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(Z)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(K)-\mu}{\sigma}-\sigma\right)\right]-$$

$$K\Bigg(A\%\left\{\Phi\left(\frac{\ln\infty-\mu}{\sigma}\right)-\Phi\left(\frac{\ln X-\mu}{\sigma}\right)\right\}+B\%\left\{\Phi\left(\frac{\ln X-\mu}{\sigma}\right)-\right.$$

$$\left.\Phi\left(\frac{\ln Y-\mu}{\sigma}\right)\right\}+C\%\left\{\Phi\left(\frac{\ln Y-\mu}{\sigma}\right)-\Phi\left(\frac{\ln Z-\mu}{\sigma}\right)\right\}+D\%\left\{\Phi\left(\frac{\ln Z-\mu}{\sigma}\right)-\right.$$

$$\left.\left.\Phi\left(\frac{\ln K-\mu}{\sigma}\right)\right\}\Bigg)\Bigg]$$

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$$\begin{aligned}\text{Fair value} = e^{-rT}\Bigg[& A\%e^{\mu+\frac{1}{2}\sigma^2}\left[1-\Phi\left(\frac{\ln(X)-\mu}{\sigma}-\sigma\right)\right]+ \\ & B\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(X)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(Y)-\mu}{\sigma}-\sigma\right)\right]+ \\ & C\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(Y)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(Z)-\mu}{\sigma}-\sigma\right)\right]+ \\ & D\%e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(\frac{\ln(Z)-\mu}{\sigma}-\sigma\right)-\Phi\left(\frac{\ln(K)-\mu}{\sigma}-\sigma\right)\right]- \\ & K\Bigg(A\%\left\{1-\Phi\left(\frac{\ln X-\mu}{\sigma}\right)\right\}+B\%\left\{\Phi\left(\frac{\ln X-\mu}{\sigma}\right)-\right. \\ & \left.\Phi\left(\frac{\ln Y-\mu}{\sigma}\right)\right\}+C\%\left\{\Phi\left(\frac{\ln Y-\mu}{\sigma}\right)-\Phi\left(\frac{\ln Z-\mu}{\sigma}\right)\right\}+ \\ & D\%\left\{\Phi\left(\frac{\ln Z-\mu}{\sigma}\right)-\Phi\left(\frac{\ln K-\mu}{\sigma}\right)\right\}\Bigg)\Bigg]\end{aligned}$$

Applying the First Principles Approach

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The fair value of the ESOP should be ***below*** that calculated using the Black-Scholes formula when

$$100\% \geq A > B > C > D \geq 0$$

The fair value of the ESOP should be ***equal*** to that calculated using the Black-Scholes formula when

$$A = B = C = D = 1$$

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Thank You!

CPD Question 1

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Which of these are market-linked performance vesting conditions?

A	An employee should be on the payroll of the company as on the vesting date
B	An employee should be rated 3 and above throughout the vesting period
C	The share price should grow by 15% p.a. over the vesting period
D	The sales of the company should grow by 10% p.a. over the vesting period

CPD Question 2

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Which of these are **NOT** needed to determine the share price distribution parameters under the Black-Scholes model?

A	The proportion of free float in the market
B	The volatility of share price returns
C	The dividend yield on the shares
D	The risk-free interest rate